Question	Scheme	Marks	AOs
1 (a)	$\mathrm{f}(x) = -3x^2 + 12x + 8 = -3(x \pm 2)^2 + \ldots$	M1	1.1b
	$= -3(x-2)^2 + \ldots$	A1	1.1b
	$= -3(x-2)^2 + 20$	A1	1.1b
		(3)	
(b)	Coordinates of $M = (2, 20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2 + 12x + 8 \, \mathrm{d}x = -x^3 + 6x^2 + 8x$	M1 A1	1.1b 1.1b
	Method to find $R =$ their $2 \times 20 - \int_0^2 \left(-3x^2 + 12x + 8\right) \mathrm{d}x$	M1	3.1a
	$R = 40 - \left[-2^3 + 24 + 16\right]$	dM1	1.1b
	$= 8$	A1	1.1b
		(5)	
		(10 marks)	
Alt(c)	$\int 3x^2 - 12x + 12 \, \mathrm{d}x = x^3 - 6x^2 + 12x$	M1 A1	1.1b 1.1b
	Method to find $R = \int_0^2 3x^2 - 12x + 12 \, \mathrm{d}x$	M1	3.1a
	$R = 2^3 - 6 \times 2^2 + 12 \times 2$	dM1	1.1b
	$= 8$	A1	1.1b

Notes:

(a)

M1: Attempts to take out a common factor and complete the square. Award for $-3(x \pm 2)^2 + \ldots$

Alternatively attempt to compare $-3x^2 + 12x + 8$ to $ax^2 + 2abx + ab^2 + c$ to find values of a and b

A1: Proceeds to a form $-3(x-2)^2 + \ldots$ or via comparison finds $a = -3, b = -2$

A1: $-3(x-2)^2 + 20$

(b)

B1ft: One correct coordinate

B1ft: Correct coordinates. Allow as $x = ..., y = ...$
Follow through on their $(-b, c)$

(c)

M1: Attempts to integrate. Award for any correct index

A1: $\int -3x^2 + 12x + 8 \, dx = -x^3 + 6x^2 + 8x \ (+c)$ (which may be unsimplified)

M1: Method to find area of R. Look for their $2 \times "20" - \int_0^{"2"} f(x) \, dx$

dM1: Correct application of limits on their integrated function. Their 2 must be used

A1: Shows that area of $R = 8$

Question	Scheme	Marks	AOs
2(a)	$y=\frac{1}{3}x^2-2\sqrt{x}+3 \Rightarrow \frac{dy}{dx}=\frac{2}{3}x-x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$x=4 \Rightarrow y=\frac{13}{3}$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=4}=\frac{2}{3}\times 4-4^{-\frac{1}{2}}\left(=\frac{13}{6}\right)\quad \therefore y-\frac{13}{3}=\frac{13}{6}(x-4)$	M1	2.1
	$13x-6y-26=0$*	A1*	1.1b
		(5)	
(b)	$\int\left(\frac{x^2}{3}-2\sqrt{x}+3\right)dx=\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x(+c)$	M1 A1	1.1b 1.1b
	$y=0 \Rightarrow x=2$	B1	2.2a
	Area of R is $\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x\right]_0^4-\frac{1}{2}\times(4-"2")\times"\frac{13}{3}"=\frac{76}{9}-\frac{13}{3}$	M1	3.1a
	$=\frac{37}{9}$	A1	1.1b
		(5)	
			(10 marks)

Notes

(a) **Calculators: If no algebraic differentiation seen then maximum in a) is M0A0B1M1A0***

M1: $x^n \to x^{n-1}$ seen at least once $...x^2 \to ...x^1$, $...x^{\frac{1}{2}} \to ...x^{-\frac{1}{2}}$, $3 \to 0$.

Also accept on sight of eg $...x^{\frac{1}{2}} \to ...x^{\frac{1}{2}-1}$

A1: $\frac{2}{3}x-x^{-\frac{1}{2}}$ or any unsimplified equivalent (indices must be processed) accept the use of $0.\dot{6}x$ but not rounded or ambiguous values eg $0.6x$ or eg $0.66....x$

B1: Correct y coordinate of P. May be seen embedded in an attempt of the equation of l

M1: Fully correct strategy for an equation for l. Look for $y-"\frac{13}{3}"="\frac{13}{6}"(x-4)$ where their $\frac{13}{6}$ is from differentiating the equation of the curve and substituting in $x=4$ into their $\frac{dy}{dx}$ and the y coordinate is from substituting $x=4$ into the given equation.

If they use $y=mx+c$ they must proceed as far as $c=...$ to score this mark.

Do not allow this mark if they use a perpendicular gradient.

A1*: Obtains the printed answer with no errors.

(b) **Calculators: If no algebraic integration seen then maximum in b) is M0A0B1M1A0**

M1: $x^n \to x^{n+1}$ seen at least once. Eg $...x^2 \to ...x^3$, $...x^{\frac{1}{2}} \to ...x^{\frac{3}{2}}$, $3 \to 3x^1$. Allow eg $...x^2 \to ...x^{2+1}$ The $+c$ is not a valid term for this mark.

A1: $\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x$ or any unsimplified equivalent (indices must be processed) accept the use of exact decimals for $\frac{1}{9}$ $(0.\dot{1})$ and $-\frac{4}{3}$ $(-1.\dot{3})$ but not rounded or ambiguous values.

B1: Deduces the correct value for x for the intersection of l with the x-axis. May be seen indicated on Figure 2.

M1: Fully correct strategy for the area. This needs to include

- a correct attempt at the area of the triangle using their values (**could use integration**)
- a correct attempt at the area under the curve using 0 and 4 in their integrated expression
- the two values subtracted.

Be aware of those who mix up using the y-coordinate of P and the gradient at P which is M0. The values embedded in an expression is sufficient to score this mark.

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.111…. isw after a correct answer

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Be aware of other strategies to find the area *R*

eg Finding the area under the curve between 0 and 2 and then the difference between the curve and the straight line between 2 and 4:

$$\int_0^2 \frac{x^2}{3}-2\sqrt{x}+3\ \mathrm{d}x+\int_2^4 \frac{x^2}{3}-2\sqrt{x}-\frac{13}{6}x+\frac{22}{3}\ \mathrm{d}x$$

M1 $x^n \to x^{n+1}$ seen at least once on either integral (or on the equation of the line $y=\frac{1}{3}x+3$)

A1 for correct integration of **either** integral $\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x$ or $\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}-\frac{13}{12}x^2+\frac{22}{3}x$ (may be unsimplified/uncollected terms but the indices must be processed with/without the $+C$)

B1 Correct value for x can be seen from the top of the first integral (or bottom value of the second integral)

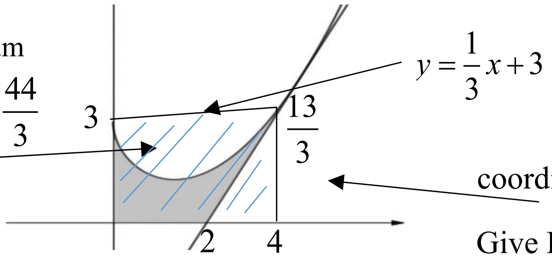
M1 Correct strategy for the area eg.

$$\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x\right]_0^2+\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}-\frac{13}{12}x^2+\frac{22}{3}x\right]_2^4=\frac{62}{9}-\frac{4}{3}(2)^{\frac{3}{2}}+\frac{76}{9}-\frac{101}{9}+\frac{4}{3}(2)^{\frac{3}{2}}$$

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.1 or 4.111….

You could also see use of the area of a trapezium and/or the use of the line $y=\frac{1}{3}x+3$ to find other areas which could be combined or used as part of the strategy to find *R*. Ignore areas which are not used. The marks should still be able to be applied as per the scheme

Area of large trapezium $=\frac{1}{2}\times\left(3+\frac{13}{3}\right)\times 4=\frac{44}{3}$



Area of triangle and the y coordinate of point P are both $\frac{13}{3}$

Give BOD if in doubt of intention

Area of trapezium – (Area between $y=\frac{1}{3}x+3$ and curve C + area of triangle)

$$=\frac{44}{3}-\frac{56}{9}-\frac{13}{3}=\frac{37}{9}$$

Question	Scheme	Marks	AOs
3	States or uses the upper limit is $\sqrt{5}$	B1	1.1b
	$\int 4x^2 + 3 \,dx = \frac{4}{3}x^3 + 3x$	M1 A1	1.1b 1.1b
	Full method of finding the area of R e.g. $23\sqrt{5} - \left[\frac{4}{3}x^3 + 3x\right]_0^{\sqrt{5}} = ...$ e.g. $\left[20x - \frac{4}{3}x^3\right]_0^{\sqrt{5}} = ...$	M1	2.1
	$\Rightarrow$ Area $R = \frac{40}{3}\sqrt{5}$	A1	1.1b
		(5)	
		(5 marks)	

Notes:

B1: States or uses the upper limit $\sqrt{5}$ Score when seen as the solution $x = \sqrt{5}$

M1: Attempts to integrate $4x^2 + 3$ **or** $\pm\left(23 - \left(4x^2 + 3\right)\right)$ which may be simplified.

Look for one term from $4x^2 + 3$ with $x^n \to x^{n+1}$ It is not sufficient just to integrate 23.

A1: Correct integration. Ignore any $+c$ or spurious integral signs. Indices must be processed.

Look for $\int 4x^2 + 3 \{dx\} = \frac{4}{3}x^3 + 3x$ **or** $\pm\int 20 - 4x^2 \{dx\} = \pm\left(20x - \frac{4}{3}x^3\right)$ if (line – curve) or (curve – line) used.

M1: Full and complete method to find the area of R including the substitution of their upper limit.
The upper limit must come from an attempt to solve $4x^2 + 3 = 23$
The lower limit might not be seen but if seen it should be 0.
See scheme for two possible ways. Condone a sign slip if (line – curve) or (curve – line) used.

A1: $\frac{40}{3}\sqrt{5}$ following correct algebraic integration.

If using (curve – line) then allow recovery but they must make the $-\frac{40}{3}\sqrt{5}$ positive.

Alternative using $\int x\,dy$

B1: States or uses limits 3 and 23. It must be for a clear attempt to integrate with respect to y

M1: Attempts to rearrange to $x =$ and integrate $\sqrt{\frac{y-3}{4}}$ condoning slips on the rearrangement.

Look for $...(y \pm 3)^{\frac{1}{2}} \to ...(y \pm 3)^{\frac{3}{2}}$

A1: Correct integration $\int \frac{(y-3)^{\frac{1}{2}}}{2} \{dy\} = \frac{1}{3}(y-3)^{\frac{3}{2}}$ Ignore any $+c$ or spurious integral signs.

M1:	Full and complete method to find the area of R including the substitution of their limits. In this case it would be for substituting 23 and 3 and subtracting either way round into their changed expression in terms of y
A1:	$\frac{40}{3}\sqrt{5}$ following correct algebraic integration.

Question	Scheme	Marks	AOs
4(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \frac{dy}{dx} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5} = 3\times5^2 - 20\times5 + 27 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 5)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
		(4)	
(b)	Both C and l pass through (0, −23) and so C meets l again on the y-axis	B1	2.2a
		(1)	
(c)	$\pm\int \left(x^3 - 10x^2 + 27x - 23 - (2x - 23)\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right]_0^5$ $= \left(\frac{625}{4} - \frac{1250}{3} + \frac{625}{2}\right)(-0)$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative:		
	$\pm\int \left(x^3 - 10x^2 + 27x - 23\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right]_0^5 + \frac{1}{2}\times5(23+13)$ $= -\frac{455}{12} + 90$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
			(9 marks)

Notes
(a) B1: Correct derivative M1: Substitutes $x = 5$ into their derivative. This may be implied by their value for $\frac{dy}{dx}$ M1: Fully correct straight line method using (5, – 13) and their $\frac{dy}{dx}$ at $x = 5$ A1: cao. Must see the full equation in the required form. (b) B1: Makes a suitable deduction. Alternative via equating l and C and factorising e.g. $x^3 - 10x^2 + 27x - 23 = 2x - 23$ $x^3 - 10x^2 + 25x = 0$ $x\left(x^2 - 10x + 25\right) = 0 \Rightarrow x = 0$ So they meet on the y-axis (c) M1: For an attempt to integrate $x^n \to x^{n+1}$ for ±" $C - l$" A1ft: Correct integration in any form which may be simplified or unsimplified. (follow through their equation from (a)) If they attempt as 2 separate integrals e.g. $\int \left(x^3 - 10x^2 + 27x - 23\right) dx - \int \left(2x - 23\right) dx$ then award this mark for the correct integration of the curve as in the alternative. If they combine the curve with the line first then the subsequent integration must be correct or a correct ft for their line and allow for ±" $C - l$" dM1: Fully correct strategy for the area. Award for use of 5 as the limit and condone the omission of the "– 0". **Depends on the first method mark.** A1: Correct exact value **Alternative:** M1: For an attempt to integrate $x^n \to x^{n+1}$ for ±C A1: Correct integration for ±C dM1: Fully correct strategy for the area e.g. correctly attempts the area of the trapezium and subtracts the area enclosed between the curve and the x-axis. Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the trapezium **and** the subtraction. May see the trapezium area attempted as $\int \left(2x - 23\right) dx$ in which case the integration and use of the limits needs to be correct or correct follow through for their straight line equation. **Depends on the first method mark.** A1: Correct exact value Note if they do $l - C$ rather than $C - l$ and the working is otherwise correct allow full marks if their final answer is given as a positive value. E.g. correct work with $l - C$ leading to $-\frac{625}{12}$ and then e.g. hence area is $\frac{625}{12}$ is acceptable for full marks. If the answer is left as $-\frac{625}{12}$ then score A0